FOURIER ANALYSIS OF THERMOACOUSTIC INSTABILITY IN LEAN COMBUSTION ENGINES

Ian Kung
Advisor: Dr. Sarah Day
William & Mary, Department of Mathematics, Williamsburg, VA 23187
Special thanks to Dr. Daniel Paxson, NASA Glenn Research Center

***Abstract*—As innovation in flight continues to expand and attract commercial and research interest, it is essential to understand unpredictable elements in combustion mechanics closely tied to the longevity of propulsion systems. Past examples and current efforts prioritized by NASA demonstrate the importance of utilizing computational methods to address unpredictability in modern propulsion methods. In particular, research in thermoacoustic instability seeks to mitigate the perturbations which arise from a combination of the combined heat release and acoustic waves in the combustion process. Thermoacoustic instability can lead to functionality concerns in gas turbines, engines and rockets. This topic has increasing relevance to the development of propulsion systems, which frequently employ lean-burning combustion processes. By incorporating greater amounts of air, lean combustion processes have the dual purpose of enhancing performance while also decreasing emissions, but are especially susceptible to instabilities in the form of high-pressure oscillations. This project focuses on the use of Fourier analysis techniques to characterize these oscillations in combustion simulations when changing the exit area opening size, reaction rate, and fuel flow rate. Analysis found relatively straightforward results when variables were manipulated independently, and observed more interesting non-linear behavior when altered together.**

I. Introduction

Unstable behavior in engineered systems is generally best avoided or reduced. Especially in power generation processes within gas turbines and combustors, vibrations and resulting heat transfer can quickly become problematic[2–6]. Previous studies have developed promising mitigation strategies through active control methods and have provided numerous computational approaches to anticipate instabilities[2–4,6–8,12]. They can also be passively avoided by redesigning the geometry of a combustor, but in actual implementations this case is expensive and less practical. A popular active control approach is to control part of the system, such as modulating fuel flow. In the context of the relevance of these changes to NASA's aeronautics directorate, modern aircraft combustors that implement premixed flow are known to result in unpredictable heat release patterns accompanied by vibration-causing acoustics[6,9,11]. This simulator is a crude approximation of a real aircraft combustor where the fuel injector introduces premixed flow. The fuel-air mixture then reaches the combustion chamber further downstream, sparking the actual combustion process and generating power/thrust.

In this project, data was obtained using the NASA Sectored One-Dimensional (S1D) Combustor Simulation[1,9,11] to effectively capture acoustic perturbations resulting from injecting premixed fuel. While lean burning combustors are prone to problematic vibrations, pressure readings themselves are ultimately self-limiting. This means pressure magnitude remains more or less constant in the long run, but can lead to issues in practice when pressure fluctuations surpass the physical limits of the combustor[2,3,5]. When instabilities are mentioned, it is referring to the long-term magnitude or size of these pressure measurements. For the most part, the default recommended parameters were used to configure the combustor. Pressure signals were quantified in MATLAB using Fourier transformations to obtain the dominant pressure frequencies, and in this way can describe the long-term pressure magnitude and frequency results given by the combustor in repeated trials. Under the right conditions, the simulator's pressure readings grow larger until they settle at a particular magnitude and frequency.

II. Simulator Configuration

The S1D combustor simulation was obtained through the NASA open source software catalog, which includes all of the accompanying documentation and source files[11]. The simulation itself is comprised of three FORTRAN 77 source code files which must be compiled to create the simulator program. It obtains its starting state from the input file flametub.dat. The included S1D Configuration program simplifies the configuration process and automatically edits the source code to the desired parameters based on user-inputted settings. Following the model detailed in the user manual and the configuration guide included with the coding package, the simulated combustor has three sectors represented by the physical structure shown below in Figure 1. The combustor is comprised of 200 numerical cells, which each record data on the pressure, velocity, temperature, and reactant fraction of the system at that point in the combustor. The cell numbers increase as you move downstream or towards the right end of the combustor. For the measurements presented in this paper, measurements are taken from cell 99.

The simulator outputs two files: flametub.new and out.dat. The first stores the last state of the previously run simulation, which allows for convenient "real time" adjustments to the combustor by updating the simulator input file flametub.dat with flametub.new. The out.dat file holds one-dimensional data for four variables across the time period specified in the simulation.

The following parameters were used to configure the simulation and remained at these values across all trials:

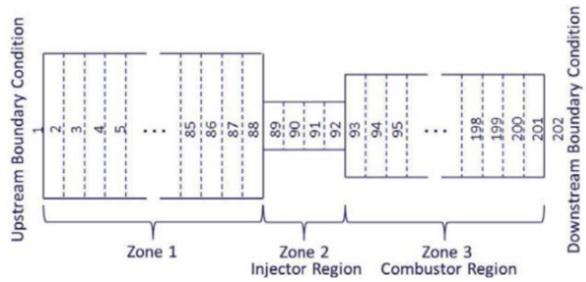

Fig. 1: 3-sectored combustor configuration with 202 cells, from "NASA One-Dimensional Combustor Simulation-User Manual for S1D_ML," 2014[11]

Configuration Parameters

Parameter Type	Value
Output frequency	5000 Hz
Combustor length	5.453 ft.
Speed of sound	1809 ft/sec
Number of sectors	3
Flameholder index	97
Fuel injector index	89
Noise index	89
Upstream Pressure Index	117
Downstream Pressure Index	176
lpcnt	26
df1	0.9403
cf0	0.9701
cf1	-0.9701

The bottom four parameters beginning with lpcnt are the noise filter parameters for the simulator. Sector changes were placed at cells 91 and 87, and the the sector area ratio option was chosen to designate a contraction and expansion in the combustor geometry. Specifically, the area ratio at index 91 was $AR1 = 0.208125$, and $AR2 = 0.029444$ at index 87. Referencing Figure 1, these area ratios translate to dividing the smaller area at the transition points over the larger area. An expansion occurs at the point referenced with $AR1$, and a contraction at the point with $AR2$.

The role of the fuel injector is to ensure the simulation resembles a practical combustor. The noise feature is closely related to the fuel injector and has a considerable impact on the obtained measurements. As advised in the configuration guide, noise was placed nearby the fuel injector and distributed over no more than two cells (one cell in this case). When introduced to the combustor simulation using the perturbation feature in the simulation, noise allows the results to more closely resemble what would be observed in a similar physical experiment. All analyses after simulation trials were done in MATLAB.

III. Simulation Procedure

As previously stated, pressure served as the form of measurement. To examine the impact of changing elements of the combustor, four parameters were varied to compare their outputs. They are 1) perturbation level 2) exit area opening size 3) reaction rate and 4) fuel flow rate. These variables were manipulated separately and then pairwise together to discern the magnitude of unstable behavior in the combustor as important variables were changed. It is important to note that the simulator yields deterministic results. With the sample input file and configuration parameters, the simulator creates identical results when run multiple times as opposed to having an uncontrollable element of randomness as would be seen in multiple physical trials. In addition, the simulator's output is dominated by a single frequency, or in other words the peaks in pressure readings occur at a regular time interval.

Comparisons between different configurations were done beginning at the time step at where asymptotic behavior begins. To select a time where pressure settles at its limit, a particular simulation where pressure readings grow uniformly was used an idealized case. This reference point had the following key parameter values which will be referred to as the default settings for the variables that are later changed independently to observe differences in their impacts on pressure values:

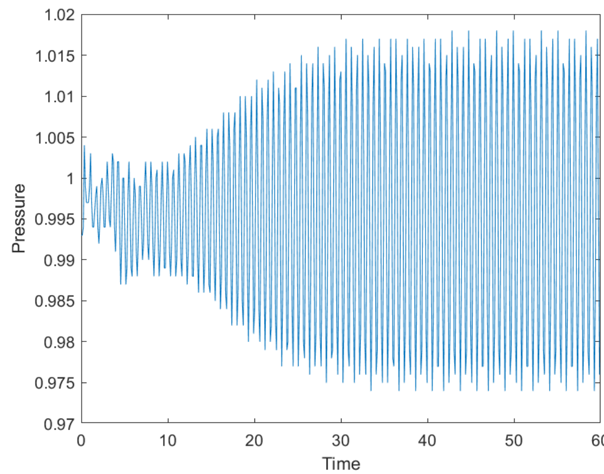

Fig. 2: Pressure readings from idealized simulation scenario

a) Perturbation/noise level = 0.0
b) Exit area opening size = 0.105
c) Reaction rate constant = 25.0
d) Fuel flow rate = 35.8

These settings were used in an idealized case where the instability develops uniformly. The amount of noise introduced by setting a higher perturbation level is meant to "muddy up" the simulator output so that it more closely resembles what would be seen in a trial for a real combustor. Setting it to zero for the base case likens it to theoretical behavior. In addition, simulations that exhibited unstable behavior gradually grew in pressure magnitude until they reached a long-term state. It is easiest to observe this time-dependent effect at times around 30 time units or longer. The simulation allows the user to specify a time to begin recording outputs, as well as the total run time. Using details from the accompanying user manual, it was calculated that 30 time units corresponds to 0.09 seconds. This base case output is shown in Figure 2 for simulation times 0 to 60. Another view of the inner workings within the combustor is included in Figure 3 as a heat map of pressure and temperature at the same settings for times 0 to 30.

Rather than comparing the pressure outputs

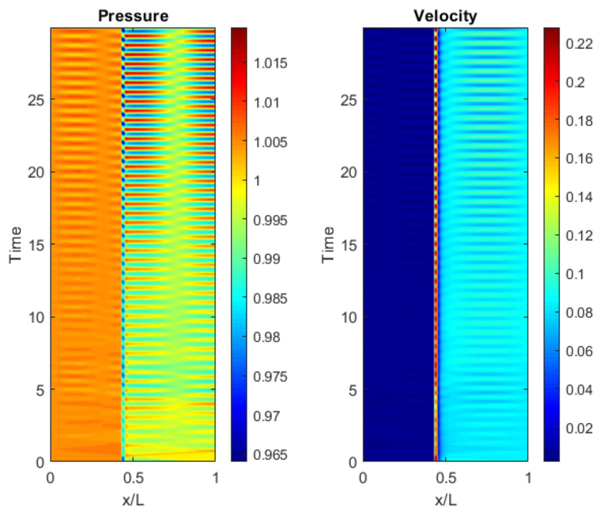

Fig. 3: 1-dimensional pressure and velocity data along time

for different combustor configurations across all time points, it was necessary to determine a time at which long term asymptotic behavior begins for consistent analysis. The pressure data clearly does not reach the long-term magnitude before 20 time units(see Figure 2), and therefore the search began at t=20. A series of Fourier transforms were then conducted to compare the dominant frequencies between different simulations after pressure readings have already settled at their long-term patterns. Fourier transforms are a well-known technique in the realm of signal processing which decomposes a potentially noisy input signal into its frequency components. More specifically, it effectively translates a complicated signal into a theoretically unlimited number of sine and cosine waves with varying amplitudes and frequencies. The starting point of asymptotic behavior was obtained using the following process:

1) Conduct simulations with time intervals 10 time steps each, from 20–30 to 60–70 with the perturbation level set to 0.

   For each interval:
2) Extract pressure data vector at cell 99, a point slightly downstream from the fuel injector.
3) Conduct a fast Fourier transform (fft()) on the vector, which outputs a complex-valued vector representing the frequency components of the input data.
4) Set the first cell of data to zero, which records the 0Hz frequency and would be disproportionally large.
5) Find the distance between adjacent time periods by calculating the norm between the absolute value of the Fourier coefficients, which gives the distance of each point in the complex plane. The results are shown in the table below.

Time Periods Compared	Norm of Difference
$[20,30],[30,40]$	0.9513
$[30,40],[40,50]$	0.3762
$[40,50],[50,60]$	0.3065
$[50,60],[60,70]$	0.2954
$[60,70],[70,80]$	0.3108

This preliminary process revealed that the differences between Fourier coefficients from adjacent time periods quickly begins to decrease, leveling out at a value of approximately 0.3. Based on these observations, the earliest time where long-term pressure readings begin was selected as 40. This time represents the point at which frequency components of the pressure data no longer vary to an extent that is meaningful for analysis. Based on the recommended length of time to record data in the simulator documentation, the analysis is based on measurement taken across 20 time units (t=40-60).

Next, the following procedure was used to quantify the nature of pressure data at different parameter settings. These steps take place after a fast Fourier transform has been conducted on the pressure data.

1) Set the first cell of data to zero, which records the 0Hz frequency and would be disproportionally large.
2) Plot the absolute value of the pressure Fourier coefficients, giving the distance of each point in the complex plane.
3) Mark the locations of the two resulting peaks, representing the positive and neg-

ative versions of the dominant frequency.
4) Set all points excluding the peaks to zero.
5) Conduct an inverse fast Fourier transform (ifft()) on the Fourier coefficients, now only two entries, to extract the sinusoidal signal.

The following snippet of code expedites this process by taking the complex vector of Fourier coefficients and outputting a real-valued vector of all zeros except for the dominant frequency (largest absolute value) at its original index.

```
function result = zero_except_max(vec)
    vec(1) = 0;
    result = zeros(size(vec));

    max_val = max(abs(vec));

    result(abs(vec) == max_val) = max_val;
end
```

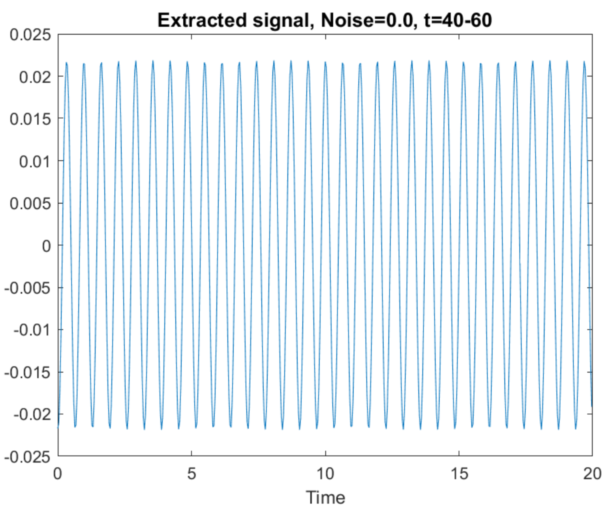

Fig. 4: Extracted signal with default settings, noise level = 0.0

IV. Findings

A. Controlling Individual Variables

After proceeding through the above steps, changes to the perturbation level were implemented to examine the differences in the extracted sinusoidal signal. Firstly, changes in perturbation level had little to no effect on the underlying signal. According to the provided configuration guide, values above 0.85 are likely unreasonable when it comes to creating output that emulates measurements from a real physical combustor test. Therefore, simulations were run with perturbation levels of 0, 0.5, and 0.8. In cases with default settings which differed only by perturbation level, the signal shown in Figure 4 was extracted.

It makes sense that the same signal would be extracted from trials that differ only by noise level, since the differences in noise are precisely what is filtered out in the Fourier transform. It should be noted that time values plotted along the horizontal axis represent the time since the simulation began recording data. That is, the time that had elapsed since the simulation has already been running for 40 time units.

More variability was observed in the Fourier coefficients of importance between trials with different exit area opening sizes. The norm/-magnitude of the differences between the base case and trials for various exit area opening sizes is shown in Figure 5. The magnitude represented on the vertical axis on the following graphs and plots represents the norm of the difference between each plotted trial and the base case using the time period of t=40-60 time units. While this magnitude is defined as the distance between the Fourier coefficients of a trial and the base case, it can be more useful to think of the magnitude value as the intensity of unstable behavior with specific combustor variable settings. Each data point observed for a particular variable was calculated by changing only the variable of interest, while the other variables remained set at their default value. For example, all data points in Figure 5 were run with noise level at 0.5, reaction rate of 25, and fuel flow rate of 35.8. The overall plot for variations in exit area opening size suggests that there is a negative relationship between the exit area size and the magnitude of unstable behavior in the combustor, which levels out at a magnitude of approximately 8. The steep dip at the center of the graph can be explained by the properties of

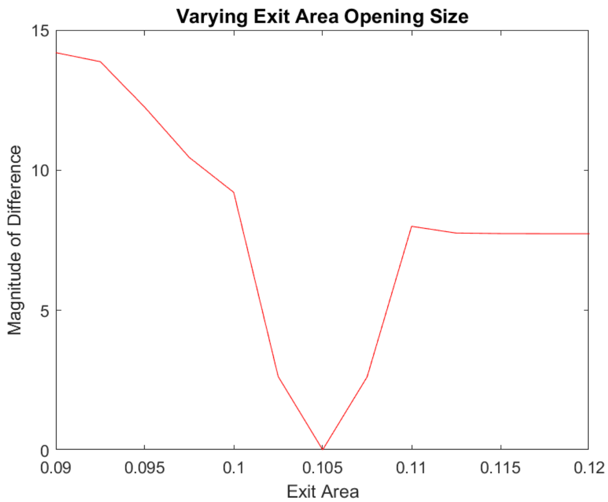

Fig. 5: Decreasing trend with increase in exit area size

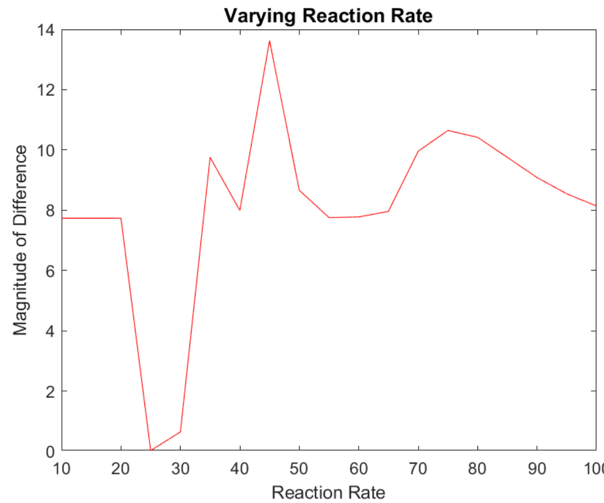

Fig. 6: Reaction rate has more unpredictable relationship to long-term pressure

the Fourier transform. Since the reference case has identical parameter settings to the trials shown except for exit area opening size, values closer to the default exit area size of 0.105 lead to a significantly smaller value. Specifically, the magnitude of the difference at the default exit area 0.105 is 0.0246, again demonstrating a minimal difference in the underlying signal between trials with differing noise/perturbation levels but the same settings otherwise. Moving further outwards from the default exit area size, Trials within 0.025 units of the default value of 0.105 have a fundamental signals that are especially similar to the base case.

Analysis of the magnitude of difference between Fourier coefficients with different reaction rates shows a less clear relationship between reaction rate and the fundamental underlying pressure signal. However, we still see the expected negligible norm of differences between the measured case and reference case at the default reaction rate of 25. Interestingly, a reaction rate of 30 has a more similar fundamental signal to the reference case than a reaction rate of 20 which results in a much larger difference despite being the same distance away from 25.

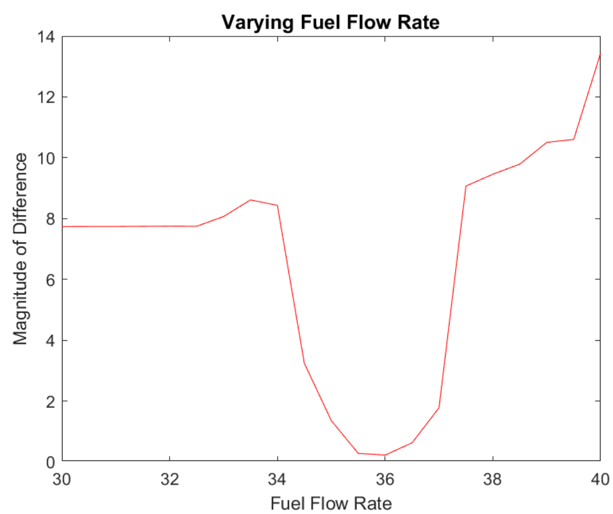

Fig. 7: Increasing size of instability with broad dip close to reference case

B. *Controlling Variable Pairs*

Somewhat of an opposite relationship exists when comparing the effect of fuel flow rate and exit area opening size. That is, while there is an expected dip in the fuel flow rate plot, the magnitude of Fourier coefficient differences seems to be gradually rising with higher fuel flow rates rather than decreasing as seen when exit area opening size is increased. There is also a visibly wide range of fuel flow rate values that more closely resemble the base

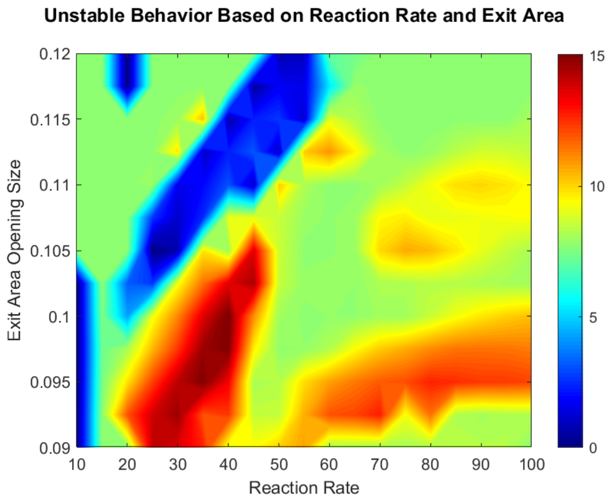

Fig. 8: Extreme differences in dominant Fourier coefficients are sensitive to reaction rate and exit area values

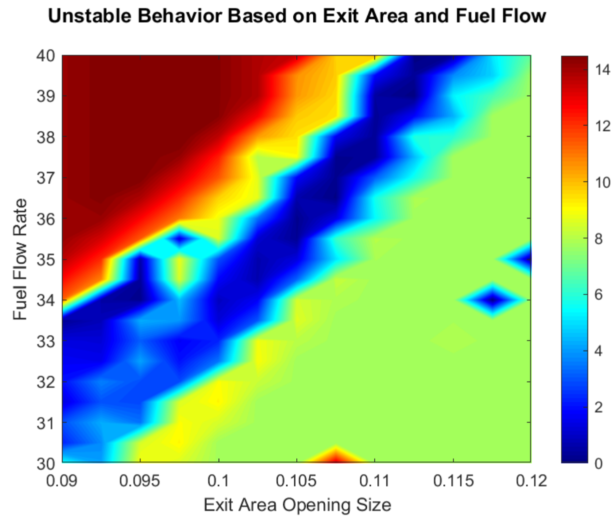

Fig. 9: Proportional relationship between exit area and fuel flow

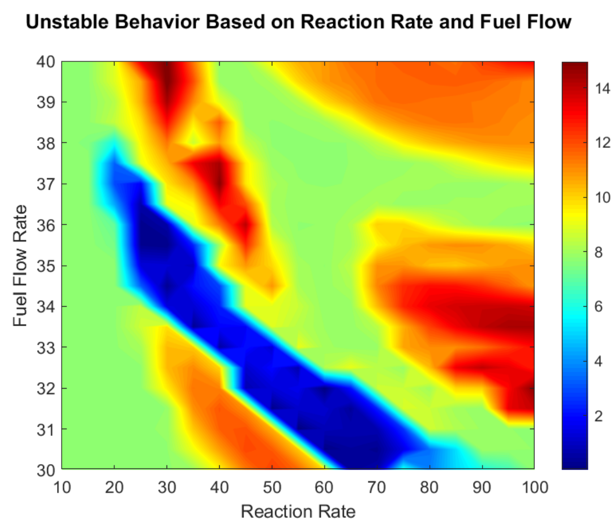

Fig. 10: High and low magnitude values are close in proximity

case. Overall, the single-variable plots show a positive relationship for fuel flow rate, negative relationship for exit area opening size, and an indeterminate affect of reaction rate on the magnitude of unstable behavior.

To extend beyond the observed measurements of unstable behavior when a single variable is altered, the procedure was repeated for the 919 possible pairwise combinations of the three variables where both chosen variables were manipulated while the third remained at its default value. Below are the resulting heatmaps of the procedure done in MATLAB.

The observed patterns in scenarios where two variables are changed simultaneously are more complex compared to varying each of the three variables individually while all other settings are set at default values. The heatmaps indicate significant interactions between each chosen pair of variables, as the magnitude of the the instability measured in the combustor is nonlinear and harder to determine when two variables are manipulated simultaneously. However, it is possible to discern some relationships between variable pairs, most notably between exit area and fuel flow where a low magnitude can be obtained when fuel flow rate is approximately 340 times the exit area opening size.

It is also important to address the color scale given by the heat maps. A low magnitude of differences is still interpreted as having a fundamental pressure signal that resembles the base case. While the range of possible magnitudes are similar in all three heatmaps, further work is required to determine whether a particular magnitude of underlying signal differences begins to cause structural problems in a physical combustor. The heat maps go so far as to suggest that certain variable configurations will cause pressure to stray further from an ideal case.

V. Conclusion

Instabilities continue to be an important aspect of lean combustion engines that are in prevalent use today. To better understand pressure fluctuations that accompany unwanted vibrations from heat release, differences between a theoretical base case and trials that emulate readings that would be seen in physical tests were compared using the Fourier transform. Using the Sectored One-Dimensional Combustion Simulator from NASA, the impact of manipulations of key variables on long-term instabilities were analyzed and compared by the underlying signal of their outputs.

The variables of exit area opening size, reaction rate, and fuel flow rate were changed first independently and then in pairs. For each combination of variables, an approximation of a real-world combustor was configured and run to measure the magnitude of the difference between fundamental pressure signals. In the one-dimensional case, relatively straightforward monotone results were obtained. When two variables were changed together, more interesting nonlinear relationships depending on the variable pair were observed. Together, these findings indicate that particular configurations are likely to lead to pressure fluctuations with a larger difference in its fundamental signal compared to a theoretical base case.

VI. Future Research

The most accessible extension of this project would be to conduct a similar analysis but for the remaining variables in the output of the simulator. They are density, velocity, and the reactant fraction (as well as temperature which can be found by dividing pressure by density). Results from separate analyses for each of these variables would certainly be interesting to compare with the end product of this project.

Manipulating additional variables and characterizing their effect on the magnitude of pressure fluctuations would further benefit an understanding of thermoacoustic instability stemming from lean combustion. Namely, changes in geometry and fuel injector location would likely lead to significant changes in behavior for the variables of interest. Custom settings in the geometry of this simulated combustor were not included in the scope of the project as it requires a substantially higher level of experience to determine realistic changes in configuration and important aspects such as the proportional area between sectors. Similarly, if the fuel injector is moved it must be contained in a reasonably positioned injector section. As the source of premixed flow, altering its location would likely have interesting effects on the development of perturbations.

As a continuation of the progress made in this project, the project's original goal to create a surrogate model using what is known about the pressure outputs from the system remains an opportunity to apply concepts from dynamical systems modeling to extract data from simulated unstable behavior and create an approximation of global dynamics.

References

[1] "Free Software Tools for Control Design, Simulation, and Analysis — Glenn Research Center — NASA," Glenn Research Center — NASA, Dec. 18, 2023.

[2] N. B. George, M. Raghunathan, V. R. Unni, R. I. Sujith, J. Kurths, and E. Surovyatkina, "Preventing a global transition to thermoacoustic instability by targeting local dynamics," Sci Rep, vol. 12, no. 1, p. 9305, Jun. 2022, doi: 10.1038/s41598-022-12951-6.

[3] H. Gotoda, Y. Shinoda, M. Kobayashi, Y. Okuno, and S. Tachibana, "Detection and control of combustion instability based on the concept of dynamical system theory," Phys. Rev. E, vol. 89, no. 2, p. 022910, Feb. 2014, doi: 10.1103/PhysRevE.89.022910.

[4] A. Haluszczynski and C. Räth, "Controlling nonlinear dynamical systems into arbitrary states using machine learning," Sci Rep, vol. 11, no. 1, p. 12991, Jun. 2021, doi: 10.1038/s41598-021-92244-6.

[5] J. Harbaugh, "Solving Combustion Instability and Saving America's First Trips to the Moon - NASA," Jul. 12, 2019.

[6] L. Kabiraj, A. Saurabh, P. Wahi, and R. I. Sujith, “Route to chaos for combustion instability in ducted laminar premixed flames,” Chaos: An Interdisciplinary Journal of Nonlinear Science, vol. 22, no. 2, p. 023129, Jun. 2012, doi: 10.1063/1.4718725.

[7] A. Mukhopadhyay, R. R. Chaudhari, T. Paul, S. Sen, and A. Ray, “Lean Blow-Out Prediction in Gas Turbine Combustors Using Symbolic Time Series Analysis,” Journal of Propulsion and Power, vol. 29, no. 4, pp. 950–960, Jul. 2013, doi: 10.2514/1.B34711.

[8] V. Nair, G. Thampi, and R. I. Sujith, “Intermittency route to thermoacoustic instability in turbulent combustors,” J. Fluid Mech., vol. 756, pp. 470–487, Oct. 2014, doi: 10.1017/jfm.2014.468.

[9] T. Stueber and D. Paxson, “NASA One-Dimensional Combustor Simulation-User Manual for S1D_ML,” 2014. [Online].

[10] E. Schmahl, ”Using the Fourier Transform,” 1999. [Online].

[11] “Time-Accurate, Sectored, One-Dimensional Reactive Code for Simulation, Prediction, and Control of Combustion Instabilities(LEW-17677-1) — NASA Software Catalog,” software.nasa.gov.

[12] Q. Zeng, C. Hu, H. Xu, J. Sun, X. Tan, and J. Zhu, “Prediction of acoustic pressure of thermoacoustic combustion instability based on Elman neural network,” Journal of Low Frequency Noise, Vibration and Active Control, vol. 42, no. 3, pp. 1519–1530, Sep. 2023, doi: 10.1177/14613484231152855.