NOVEL JOSEPHSON JUNCTIONS FOR CRYOGENIC MEMORY AND FAULT-TOLERANT QUANTUM COMPUTATION

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Abstract

Recent advances in fabrication techniques allow the realization of novel Josephson junctions (JJ) with exotic properties that have the potential to enhance the strength and efficiency of high-performance computing resources vital to NASA's mission and advance quantum communication efforts by NASA. Among the many applications of JJ's, Josephson mangetic random-access memory (JMRAM) is an energy-efficient, fast, and memory-dense option for cryogenic memory in classical and quantum computers. JMRAM proposals currently rely on carefully built multilayer JJ's that would be difficult to scale. Here we propose an alternative design to generate spin-polarized supercurrent by implementing a spin-triplet superconductor into the JMRAM design. We also consider JJ's that are predicted to host topological superconductivity which can be used in topological quantum computing schemes. A signature of non-trivial topology is missing odd Shapiro steps. We provide theoretical support for the first strong report of missing Shapiro steps in a topologically trivial JJ. We ascribe our observations to the high transparency of our junctions allowing Landau-Zener transitions. We analyze our results using a bi-modal transparency distribution which demonstrates that only few modes carrying 4 periodic current are sufficient to describe the disappearance of odd steps.

Introduction

Quantum computers are of current interest to NASA because of their ability to solve a certain class of problems much more efficiently than classical computers. Currently, the quantum computers that have been made are limited by a number of factors, but a particularly challenging obstacle is decoherence–the corruption of quantum information contained in a quantum bit (qubit) due to interactions with the environment. A proposed way to overcome this obstacle is by incorporating so called *topologically-protected* (TP) quantum states (states protected from weak perturbations due to the environment). A number of theoretical works[1,2] have discussed setups in which TP states can be realized and used to create a qubit that *avoids* decoherence, and the experimental community is now concerned with verifying the topological phases predicted in these theoretical works. Recently, multiple groups have produced evidence for TP quantum states in planar Josephson junctions.[3,4]

Researchers at NYU, in collaboration with our group, studied a novel JJ that they developed based on epitaxially grown Al/InAs multilayers.[5] Their junction features many conduction channels (some with very high transparency–a description of how strongly a quantum mode contributes to the total electrical current in a nanoscale circuit element); and by tuning the free-electron density (i.e. an external gate bias), the system can be more easily tuned into a topological phase where a TP qubit can be realized.[2] Among a number of ways to probe the topological phase of a junction, one way is to measure Shapiro steps– constant-voltage steps at integer multiples of $\hbar\omega_{ac}/2e$ in the I-V curve of an JJ irradiated with radiation of angular frequency ω_{ac}. Arguments for verification of topological phases include the observation of suppressed odd-integer Shapiro steps.[6]

Missing Shapiro Steps

We have reported in the journal *Nature Communications*[7] the first strong experimental observation of missing Shapiro steps in a topologically *trivial* JJ. We developed an resistively-shunted junction model where the supercurrent channel is described by a bimodal distribution of Andreev bound state transparencies where the contribution from high transparency modes is allowed to particpate in Landau-Zener transitions (LZT)–diabatic quantum transitions between energy levels.

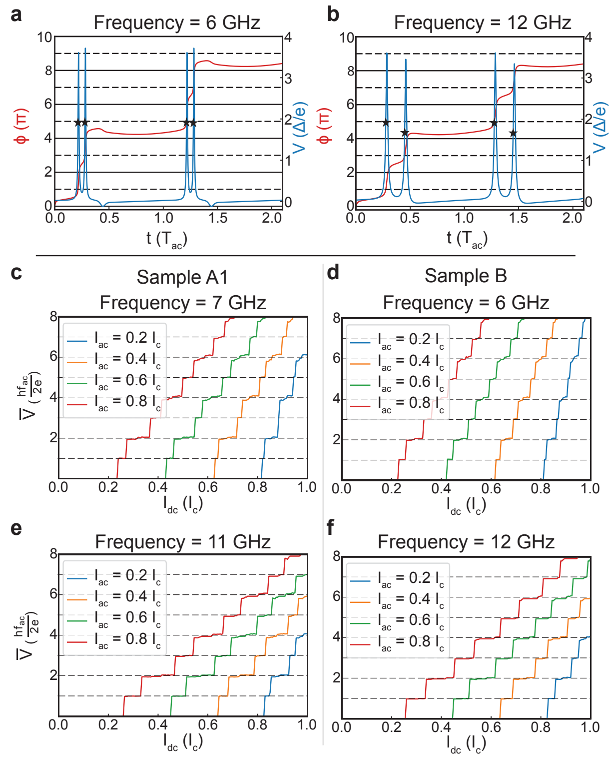

Figure 1: **a, b**: Phase and instantaneous voltage across the JJ, which corresponds to the second Shapiro step. The black stars indicate the instantaneous voltage at $\phi = \pi$ used in the calculation of the Landau-Zener transition. **c, e** $\overline{V}$-$I_{\rm dc}$ curves showing Shapiro steps at 7 and 11 GHz for the parameters of device A in the experiment. **d, f** $\overline{V}$-$I_{\rm dc}$ curves showing Shapiro steps at 6 and 12 GHz for the parameters of device B in the experiment.

Methods

The dynamics of a current-driven JJ can be treated within a semiclassical description where the current is carried in three parallel channels: a resistive channel describing dissipative current, a capacitive channel describing charge accumulation on the superconducting leads across the weak link, and a supercurrent channel describing current mediated by Cooper pairs. Our JJ has a small geometric capacitance ($C \sim 1$ fF) corresponding to the overdamped regime; thus, we neglect the capacitive channel. Driving our JJ with an ac current, we can use Kirchoff's junction law to write

$$I_{\text{drive}}(t) = I_R(\dot{\phi}) + I_s(\phi) \tag{1}$$

where ϕ is the phase difference between the two superconducting electrodes, $I_{\text{drive}}(t) = I_{\text{dc}} + I_{\text{ac}}\sin(2\pi f_{\text{ac}}t)$ is the external driving current, $I_{\text{R}}(\dot{\phi}) = \frac{\hbar}{2eR}\dot{\phi}$ is the current in the resistive channel, and $I_s(\phi)$ is the supercurrent contribution. Then the equation governing the JJ's dynamics is

$$\frac{\hbar}{2eI_cR}\dot{\phi} = \hat{I}_{\text{dc}} + \hat{I}_{\text{ac}}\sin(2\pi f_{\text{ac}}t) - \hat{I}_s(\phi), \tag{2}$$

where the hats denote currents normalized by the critical supercurrent I_c.

In JJ's, the supercurrent is mediated by Andreev bound states (ABS) with energy given by

$$E_{\text{ABS}} = \pm\Delta\sqrt{1-\tau\sin^2(\phi/2)} \tag{3}$$

where Δ is the SC gap and τ is the transparency. The spectrum according to Equation (3) exhibits a gap $2\Delta\sqrt{1-\tau}$ at $\phi = \pi$. The supercurrent carried by a single ABS at zero temperature is given by a skewed sinusoidal CPR,

$$I_{\text{ABS}} = \frac{e\Delta}{2\hbar}\frac{\tau\sin(\phi)}{\sqrt{1-\tau\sin^2(\phi/2)}}. \tag{4}$$

In what follows, we assume the distribution of mode transparencies to be bimodal, where the two lobes of the bimodal distribution curve are centered around a low transparency τ_{low} and a high transparency $\tau_{\text{high}} > \tau_{\text{low}}$. We further assume the majority of the modes reside near τ_{low}. Now, we simplify the contribution made to the supercurrent channel by letting

$$I_{\text{low}\,\tau} = \frac{e\Delta}{2\hbar}\sum_{\tau\in\mathcal{T}_{\text{low}}}\frac{\partial E_\tau}{\partial\phi} \approx nI_{2\pi}\sin(\phi) \tag{5}$$

and

$$\begin{aligned} I_{\text{high}\,\tau} &= \frac{e\Delta}{2\hbar}\sum_{\tau\in\mathcal{T}_{\text{high}}}\frac{\partial E_\tau}{\partial\phi} \\ &\approx \frac{(1-n)I_{\tau_{\text{high}}}\sin(\phi)}{\sqrt{1-\tau_{\text{high}}\sin^2(\phi/2)}} \end{aligned} \tag{6}$$

where $\mathcal{T}_{\text{low}}$ and $\mathcal{T}_{\text{high}}$ are the set of transparencies of modes belonging to the lower τ and higher τ modes, respectively and n is the fraction of critical current contributed by the effective low-transparency mode and I_c is the critical current. Thus, we consider two *effective* modes: a low-transparency mode with a sinusoidal CPR and a high-transparency mode with a skewed CPR determined by an *effective* transparency $\tau = \tau_{\text{high}}$. Then we write our supercurrent as:

$$\begin{aligned} I_s &= I_c\frac{n}{\alpha_0}\sin(\phi) \\ &+ I_c\frac{1-n}{\alpha_\tau}\frac{\sin(\phi)}{\sqrt{1-\tau\sin^2(\phi/2)}}, \end{aligned} \tag{7}$$

We have included normalizations α_0 and α_τ that are determined by,

$$\alpha_0 = \frac{1}{\sin(\tilde{\phi}_{\max})} \tag{8}$$

$$\alpha_\tau = \frac{\sqrt{1-\tau\sin^2(\tilde{\phi}_{\max})}}{\sin(\tilde{\phi}_{\max})} \tag{9}$$

where $\tilde{\phi}_{max}$ is such that

$$\begin{aligned} \max&\left(n\sin(\phi) + (1-n)\frac{\sin(\phi)}{\sqrt{1-\tau\sin^2(\phi/2)}}\right) \\ &= n\sin(\tilde{\phi}_{\max}) \\ &+(1-n)\frac{\sin(\tilde{\phi}_{\max})}{\sqrt{1-\tau\sin^2(\tilde{\phi}_{\max}/2)}} \end{aligned} \tag{10}$$

These normalizations remove the inherent difference between the sinusoidal and skewed CPR's contribution to the critical current, allowing n to accurately determine the fraction of critical current coming from the low and high transparency channels.

In a two-level quantum system, Landau-Zener processes describe diabatic energy level transitions. Generally, the Landau-Zener transition (LZT) probability will depend on the difference in energy between the two states and the rate at which the dynamical variable changes: a small energy gap and rapid evolution of the dynamical variable are favorable conditions for an LZT to occur. We can treat the ground state and excited state of a single ABS with transparency τ as a two-level quantum system and solve for the LZT probability at the avoided crossing:

$$P_{\mathrm{LZT}}(t) = \exp\left(-\pi \, \frac{\Delta(1-\tau)}{e\,|V(t)|}\right). \qquad (11)$$

Here we neglect interference effects due to phase fluctuations and coherence between LZTs.

A successful LZT will change the sign of the supercurrent contribution due to the ABS mode undergoing the transition. For high transparency modes, LZT probability can be significant because of the small gap at the avoided crossing. We model the collective behavior of the high-transparency modes by considering a single *effective* LZT in our calculations occurring at avoided crossings. Thus, we take our supercurrent to be given by,

$$\begin{aligned} I_s = I_c \frac{n}{\alpha_0} \sin(\phi) \\ + sI_c \frac{(1-n)}{\alpha_\tau} \frac{\sin(\phi)}{\sqrt{1-\tau\sin^2(\phi/2)}}, \end{aligned} \qquad (12)$$

where $s = \pm$ controls the sign flip due to an LZT. We solve Equation (2) dynamically to account for LZTs at the avoided crossing of the effective high-transparency mode.

Results

In Fig. 1(c-f) the results of the model which reproduce the experimental observations of a suppressed first step at low driving frequency are shown. We also show in Fig. 1(a-b) the very weak dependence of the instantaneous voltage at avoided crossings in the spectrum on driving frequency. Since the LZT probability occurs at avoided crossings and depends on the instantaneous voltage, this implies the LZT probability is very weakly dependent on driving frequency. This is significant because, under the assumption of a strong frequency dependence, the lower frequency at which missing steps are observed has been used to estimate the required transparency for LZTs to explain missing steps.[8] We show that observing missing steps is possible in our model with a transparency below that estimated threshold. With a better understanding of how to estimate this transparency threshold, future experiments can better determine if missing Shapiro steps are due to topological superconductivity or some other trivial mechanism.

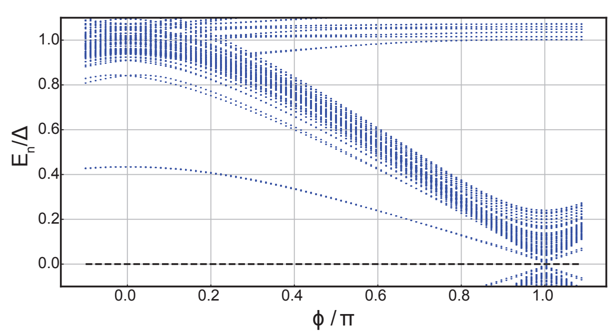

Figure 2: Andreev bound state spectrum for a wide, ballistic JJ. The calculation using tight binding parameters for InAs and Al.

We also calculated the Andreev spectrum for a Al-InAs-Al JJ using the python package Kwant[9] to better understand the Andreev spectrum on the experimental device. The result is shown in Fig. 2. For energies E below the superconducting gap

Δ, there are clearly Andreev bound states that approach the quasi-continuum ($E > \Delta$ states) near $\phi = 0$, but there are a few that lie well below the quasi-continuum which are not likely to tunnel into higher energy states. Furthermore, these states that are separated from the quasi-continuum have energies very close to zero near the avoided crossing at $\phi = \pi$ which implies that they are high transparency modes. These high transparency modes can undergo LZT's and are protected from tunneling into the quasi-continuum. We suggest that these states are responsible for the missing steps in the experiment and are unique to *wide*, ballistic JJ's.

Cryogenic Memory

In conventional superconductivity, electrons at low temperatures form Cooper pairs, where two electrons with *opposite spin and momentum* are weakly bound to each other. This type of superconductivity is called *spin-singlet* since the spinor part of the wave function corresponds to a spin-singlet state; but the Pauli exclusion principle also allows for *spin-triplet* pairing states. This exotic type of pairing has been realized in hybrid structures where multiple ferromagnetic layers (F-layers) in proximity to a conventional superconductor create a pseudospin-valve by biasing the spin-states of the Cooper pairs that coherently tunnel into the F-layers.

Researchers at Northrop Grumman Corporation have developed a memory concept called JMRAM[10] where the distinguishing feature from traditional MRAM is that JMRAM uses a JJ current measurement for the readout process. JMRAM is an energy-efficient, fast, and memory-dense alternative for cryogenic memory in *classical and quantum* computers.[10] A leading design for JMRAM uses spin-triplet JJ's that uses a hybrid structure with three F-layers (S-F1-F2-F3-S), where the first two F-layers are used to create an *equal-spin* Cooper pair–a pair or electrons with the same spin orientation that are weakly bound. An illustration is shown at the top of Fig. 3. Flipping the magnetization of either the first or last layer changes the electric current signal in the junction, which is the source of the readout process.

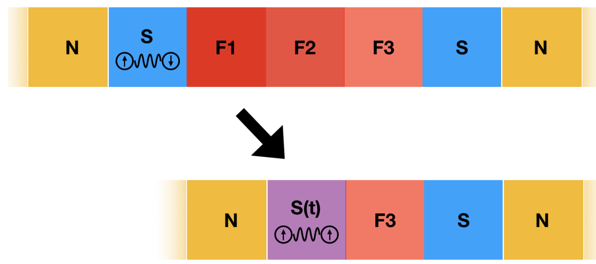

Figure 3: Illustration of the typical spin-triplet JMRAM setup (top) and my proposal to use the newly-discovered equal-spin superconductor UTe_2 (bottom).

This mesoscopic device faces challenges in scaling up production: the design requires manufacturing three F-layers with independently tunable magnetizations, and it has a smaller signal (critical current) than competing designs. Taking the concept behind the spin-triplet design, I propose to replace the junction S-F1-F2-F3-S with S_t-F-S, where S_t represents an equal-spin superconductor (a particular type of spin-triplet pairing). An illustration of the setup is shown at the bottom of Fig. 3. A group of researchers at NIST reported that UTe_2 hosts the rare and exotic equal-spin pairing,[11] which opens the door to explore the capabilities of spin-triplet superconductivity in this setup and verify the type of pairing in UTe_2. This system is also suggested to be a topological superconductor hosting chiral Majorana edge and surface states,[12] opening the door to possible applications in topological quantum computing.

Outlook

In our work[7] we show experimentally that in JJs that are undoubtedly in a topologically trivial phase, for the microwave powers and frequencies reported, there are missing odd Shapiro steps consistent with the 4π periodic current-phase relation of a topological JJ. We attribute our measurement to the very high transparency of a fraction of the modes in our JJ combined with large value of $I_c R_n$. Our results clearly show that caution should be used to attribute missing Shapiro steps to the presence of Majorana modes. They provide essential guidance to future experiments to use JJs to unambiguously establish the presence of topological superconductivity, and, in addition, significantly enhance our understanding of high quality JJs. We plan to continue working with our collaborators at NYU and investigate Shapiro steps under magnetic fields which push the device into a topological phase. We also plan to improve the models describing the AC response of JJ's with high transparency modes and investigate LZT's in these systems.

JMRAM is an energy-efficient, fast, and memory-dense alternative for cryogenic memory in classical and quantum computers.[10] We plan to conduct numerical simulations on the S_t-F-S system with realistic parameters to estimate how well this system would perform in an experiment and compare with competing designs.

References

[1] Chetan Nayak, Steven H. Simon, Ady Stern, Michael Freedman, and Sankar Das Sarma. Non-Abelian anyons and topological quantum computation. *Reviews of Modern Physics*, 2008.

[2] Falko Pientka, Anna Keselman, Erez Berg, Amir Yacoby, Ady Stern, and Bertrand I. Halperin. Topological superconductivity in a planar Josephson junction. *Physical Review X*, 2017.

[3] Hechen Ren, Falko Pientka, Sean Hart, Andrew T. Pierce, Michael Kosowsky, Lukas Lunczer, Raimund Schlereth, Benedikt Scharf, Ewelina M. Hankiewicz, Laurens W. Molenkamp, Bertrand I. Halperin, and Amir Yacoby. Topological superconductivity in a phase-controlled Josephson junction, 2019.

[4] Antonio Fornieri, Alexander M. Whiticar, F. Setiawan, Elías Portolés, Asbjørn C.C. Drachmann, Anna Keselman, Sergei Gronin, Candice Thomas, Tian Wang, Ray Kallaher, Geoffrey C. Gardner, Erez Berg, Michael J. Manfra, Ady Stern, Charles M. Marcus, and Fabrizio Nichele. Evidence of topological superconductivity in planar Josephson junctions, 2019.

[5] William Mayer, Matthieu C Dartiailh, Joseph Yuan, Kaushini S Wickramasinghe, Enrico Rossi, and Javad Shabani. Gate controlled anomalous phase shift in Al/InAs Josephson junctions. *Nature Communications*, 11(1):212, 2020.

[6] Fernando Domínguez, Fabian Hassler, and Gloria Platero. Dynamical detection of Majorana fermions in current-biased nanowires. *Physical Review B - Condensed Matter and Materials Physics*, 2012.

[7] Matthieu C Dartiailh, Joseph J Cuozzo, Bassel H Elfeky, William Mayer, Joseph Yuan, Kaushini S Wickramasinghe, Enrico Rossi, and Javad Shabani. Missing Shapiro steps in topologically trivial Josephson junction on InAs quantum well. *Nature Communications*, 12(1):78, 2021.

[8] J. Wiedenmann, E. Bocquillon, R. S. Deacon, S. Hartinger, O. Herrmann, T. M. Klapwijk, L. Maier, C. Ames, C. Brüne, C. Gould, A. Oiwa, K. Ishibashi, S. Tarucha, H. Buhmann, and L. W. Molenkamp. 4ϕ-periodic Josephson supercurrent in HgTe-based topo-

logical Josephson junctions. *Nature Communications*, 2016.

[9] Christoph W. Groth, Michael Wimmer, Anton R. Akhmerov, and Xavier Waintal. Kwant: A software package for quantum transport. *New Journal of Physics*, 2014.

[10] Ian M. Dayton, Tessandra Sage, Eric C. Gingrich, Melissa G. Loving, Thomas F. Ambrose, Nathan P. Siwak, Shawn Keebaugh, Christopher Kirby, Donald L. Miller, Anna Y. Herr, Quentin P. Herr, and Ofer Naaman. Experimental Demonstration of a Josephson Magnetic Memory Cell with a Programmable ϕ-Junction. *IEEE Magnetics Letters*, 2018.

[11] Sheng Ran, Chris Eckberg, Qing Ping Ding, Yuji Furukawa, Tristin Metz, Shanta R. Saha, I. Lin Liu, Mark Zic, Hyunsoo Kim, Johnpierre Paglione, and Nicholas P. Butch. Nearly ferromagnetic spin-triplet superconductivity. *Science*, 2019.

[12] Lin Jiao, Sean Howard, Sheng Ran, Zhenyu Wang, Jorge Olivares Rodriguez, Manfred Sigrist, Ziqiang Wang, Nicholas P. Butch, and Vidya Madhavan. Chiral superconductivity in heavy-fermion metal UTe2. *Nature*, 2020.