Abstract

NASA Langley Research Center (LaRC) and Old Dominion University (ODU) Department of Aerospace Engineering are collaborating to develop a methodology for dynamic modeling of aircraft using wind tunnel measurements. Aircraft experience unsteady and nonlinear aerodynamics that are currently misunderstood and difficult to model mathematically. NASA LaRC has developed the hardware for a dynamic test rig for using the NASA Langley 12-foot Wind Tunnel (LaRC 12-FT WT). However, software development has been fraught with difficulty. Currently the software is being worked on by a NASA engineer. The software will be uploaded, bench tested, and implemented on the larger scale dynamic rig (DR) in the LaRC 12-FT WT. Concurrently ODU is working on a model simulation of the DR in order to determine possible sources of error in the rig. The second objective is to leverage the power of Design of Experiments (DOE) and Response Surface Methodology (RSM) to exercise, validate, and examine the sources of error on the DR. These methods will also allow for optimization of the rig to be conducted. The final objective is develop a general dynamic test modeling method for an aircraft using the DR and ultimately using a DOE/RSM approach to develop the necessary empirical models of aircraft unsteady aerodynamics.

Introduction

Due to the need for increased maneuverability of modern aircraft, the aircraft will experience nonlinear and unsteady aerodynamics that is currently misunderstood and difficult to model mathematically. Take for example, when an aircraft maneuvers rapidly at high angular rates the impact of unsteady flow phenomena becomes more pronounced, as shown in Figure (1). Poor model response prediction occurs for various reasons such as: high angle-of-attack, rapid maneuvers, shock waves, separated flows, and vortical flows.

One primary shortcoming of trying to accurately model the aerodynamics is the use of the conventional stability or otherwise known as aerodynamic stability derivatives. The stability derivatives are based on the work of Bryan in 1911. The derivatives are modeled on a method that estimates the loads on the basis of instantaneous motion parameters and a linear relationship between the motion and the loads [1]. However, for modern aircraft, especially military, with highly nonlinear aerodynamic characteristics undergoing agile maneuvers at high angle-of-attack, the stability derivative model breaks down completely.

Figure 1: Demonstration of the range of unknown phenomena in flight [2].
There are several experimental test techniques used to obtain the dynamic stability derivatives. Some examples are: captive, wind-tunnel single degree-of-freedom (1-DOF), wind-tunnel free-flying and atmospheric free-flying [3]. Captive testing is composed of forced oscillation testing and rotary balance testing. This study will focus on the forced oscillation testing currently being used in NASA LaRC 12-Ft Low-Speed Wind Tunnel (LST).

Forced oscillation tests use a model rigidly mounted on a support system and then actuated to impart motion to the model while measuring forces and moments acting on the model. The forced motion has classically been sinusoidal, but due to nonlinear aerodynamic response with motion parameters, use of alternate motion shapes such as frequency sweeps or Schroeder sweeps have also been used more recently [3]. There are currently three testing configurations: roll, yaw, and pitch; as depicted in Figure (2).

The 12-Ft LST is typically used as a concept development laboratory and it provides a wide range of testing capabilities. Testing ranges from static and dynamic testing, force and moment measurements, pressure measurements, and flow visualization [3]. The tunnel can operate at dynamic pressure ranges from 0.25 to 7 psf. The rig is computer controlled and can be operated in any configuration discussed above. The rig is a hydraulically actuated system that is sting-mounted on a C-strut system; Figure (3) [3]. The arrangements rotate the model about the moment reference center of the internally mounted balance, over a total angle of attack range of about 85° [3]. It has a maximum capability of 260 °/sec pitch rate and 2290 °/sec² pitch acceleration [3]. For roll oscillation tests the maximum capability of ±170° in roll, 190 °/sec in roll rate, and 12750 °/deg² roll acceleration [3]. However it should be noted that there are maximum acceleration limits for the balance. Additional motion shapes tested other than sinusoidal oscillations include ramp motions (constant pitch rate), sum of sinusoidal motions at various frequencies and amplitudes, and Schroeder sweeps.

After testing the aerodynamic stability derivatives are calculated by various methods; such as the "integral" methods developed by Klein [4]. One common testing method traditionally runs tests at frequencies near the Dutch roll natural frequency and short period natural frequency of the aircraft over small amplitudes to estimate damping derivatives (i.e. the unsteady component of the stability derivatives) [3]. The primary averaged in-phase and out-of-phase damping coefficients from oscillation about the body axis are provided on Table (1).

The in-phase components are terms that are "in-phase" with the sinusoidal input and the out-of-phase components are terms that are "90° out-of-phase" with the sinusoidal input. Focusing on only the pitch oscillation
testing during a forced oscillation test the α and q terms cannot be separated. This is due to the fact that the angle of attack and pitch angle are physically the same during a wind tunnel test; however, this is not true in general during free-flight [2]. Therefore during testing only combined parameter can be measured.

Table 1: Primary in-phase and out-of-phase coefficients.

<table>
<thead>
<tr>
<th>Oscillation Axis</th>
<th>In-phase</th>
<th>Out-of-phase (damping)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>$C_{\alpha} = C_{\alpha} - k^2 C_{\alpha}$</td>
<td>$C_{\alpha} = C_{\alpha} + C_{\alpha}$</td>
</tr>
<tr>
<td>Roll</td>
<td>$C_{\beta} sin(\alpha) - k^2 C_{\beta}$</td>
<td>$C_{\beta} = C_{\beta} + C_{\beta} sin(\alpha)$</td>
</tr>
<tr>
<td>Yaw</td>
<td>$C_{\gamma} = C_{\gamma} cos(\alpha) + k^2 C_{\gamma}$</td>
<td>$C_{\gamma} = C_{\gamma} - C_{\gamma} cos(\alpha)$</td>
</tr>
</tbody>
</table>

Problem Statement

As discussed, the classical aerodynamic stability derivatives method assumes that there is a linear relationship between the motion and loads. This linear relationship is no longer valid during high angular rates; causing unsteady aerodynamic phenomena. The unsteady aerodynamic stability derivatives then become present in the equations of motion for an aircraft. A solution to the equations is needed to develop control laws for the aircraft. However, in order to do so the stability derivatives must be estimated. Determining the unsteady aerodynamic stability derivatives; listed on Table (1), can be determined by forced oscillation testing in a wind tunnel using data reduction techniques; such as the "integral" method [4].

However, when using the forced oscillation test the question becomes how accurate is the rig itself. For example, the engineer commands a sinusoidal input for the rig; is the rig reproducing the input waveform? If not, this would affect the offline computational method to calculate the unsteady aerodynamic stability derivatives because it assumes a "perfect" input. Another question, is the noise in the rig itself masking the aerodynamic measurement at the same frequency?

These questions can be answered by using the power of design of experiments (DOE) and response surface methods (RSM). This study has chosen to focus on longitudinal motion only (i.e. pitch oscillation testing). The study was limited to longitudinal motion because of the available information for simulation including computational codes; such as vortex lattice methods. DOE is a statistical based method to study the effect of variation on a process. RSM focuses on optimization and building higher order response models. To answer the first question, an initial characterization study of the 12-ft LST dynamic test rig can be done by using a computer simulation since the actual rig's software is currently being redeveloped for safety issues. The second question can be answered using sensitivity testing by DOE methods. As with any computer simulation the results need to be verified by experimental data and will be limited to regions of attached flow. Currently this study is focusing on using an oscillating 70° delta wing for comparison to available experimental data. Regression models will be obtained for the aerodynamic stability derivatives as a function of dynamic rig parameters such as sting inertia, model inertia, gear backlash etc.

Computer Simulation

The computer simulation of the dynamic rig during a pitch oscillation test is based on the user inputting a prescribed sinusoidal motion. A 3-phase AC induction motor powers a gear train which in turn drives a model support (sting). The system is modeled as compliantly coupled, appropriate when resonance is a problem in mechanical systems. Finally the aircraft model resists motion through aerodynamic torque. The system feeds back the motor's rotor position, velocity,
and current. The computer simulation is depicted in Figure (4).

Three-Phase AC Induction Motor

Applying the Krause model, see Figure (5), an equivalent circuit in the arbitrary frame (d-q frame) is obtained. The equations of motion for a 3-phase AC induction motor can then be derived from the equivalent circuit and placed in state-space form.

\[
\frac{d\omega_n}{dt} = \left(\frac{p}{2J}\right)\left(T_e - T_i\right) \tag{5}
\]

These equations are then used in a Simulink program along with a pulse-width modulation model and a vector control algorithm. The main objective of vector control is to independently control the developed torque and flux like a direct-current (DC) motor with separately excited states. There are two different schemes for applying vector control: the direct and the indirect scheme. The latter is used in this study. The indirect scheme simply calculates the slip between the rotor and stator. A detailed schematic of the motor is provided in Figure (6).

Pulse-width modulation (PWM) acts as an inverter for the AC motor. The inverter creates an isosceles triangle carrier wave and compares it with a fundamental-frequency sine modulating wave. The natural points of intersection determine the switching points of the power devices of a half-bridge inverter. The 3-phase voltage waves are shifted 120° to one another and thus a 3-phase motor can be supplied as shown in Figure (7).
Resonance and Compliance

Motor drives are required to have high performance, achieved by closed loop controllers; such as, PI controllers. The down side of high performance is mechanical resonance. Mechanical resonance is caused by compliance between two or more components in mechanical transmission. The resonance is typically the compliance between the motor and load. The reduced compliance model is illustrated on Figure (8). It should be noted that there is a gear ratio (1:89) between the motor and load. The reduced compliantly-coupled drivetrain model.

\[
J_{a}\theta_{a} = T_{motor} + \frac{1}{89} \left[ B_{\text{mod}} (\theta_{r} - \theta_{a}) + K_{\text{mod}} (\theta_{j} - \theta_{a} ) \right] \quad \ldots (6)
\]

\[
J_{a}\dot{\theta}_{a} = -B_{\text{mod}} (\theta_{r} - \theta_{a}) - K_{\text{mod}} (\theta_{j} - \theta_{a}) + B_{\text{mod}} \dot{\theta}_{\text{mod}} + \dot{\theta}_{j} + K_{\text{mod}} (\dot{\theta}_{\text{mod}j} - \dot{\theta}_{j}) \quad \ldots (7)
\]

\[
J_{\text{mod}} \dot{\theta}_{\text{mod}} = -B_{\text{mod}} (\dot{\theta}_{\text{mod}j} - \dot{\theta}_{j}) - K_{\text{mod}} (\dot{\theta}_{\text{mod}j} - \dot{\theta}_{j}) + T_{\text{aero}} \quad \ldots (8)
\]

Unsteady Vortex Lattice Method

The vortex lattice method (VLM) is an extension of lifting line theory. The method assumes a wing as a surface then superimposes a grid of horseshoe vortices at a specified control point location. At each control point the induced velocity is calculated using the law of Biot-Savart. A summation of all the control points on the wing is performed which produces a set of linear algebraic equations for the vortex strengths that satisfy the boundary condition of no flow through the wing. The vortex strengths are related to the wing circulation and pressure differential between the upper and lower surfaces. The pressure differentials are integrated to yield the total forces and moments. The method can handle account for unsteadiness by using the unsteady Bernoulli equation. The computational method outlined by Katz and Plotkin for an unsteady vortex lattice code was written in Matlab and validated with textbook results [5]. However, since the code is discrete it is not readily available to implement into the Simulink model which is continuous. For the first try a closed-form solution for slender delta-wing in pitch oscillation was used for the aero-model [5]. The equations for the lift and pitch moment of a slender delta-wing in pitch oscillation are listed below.

\[
L = \left[ \int_{b}^{c} \int_{0}^{\infty} \left( \frac{3}{4} \cos \theta + \frac{3}{4} \cos \theta \sin \theta \right) \sin \theta \right] \quad \ldots (9)
\]

\[
M = \left[ \int_{b}^{c} \int_{0}^{\infty} \left( \frac{3}{4} \cos \theta + \frac{3}{4} \cos \theta \sin \theta \right) \sin \theta \right] \quad \ldots (10)
\]

The overall simulation model is provided in the Appendix. The user inputs a sinusoidal input with respect to the motor’s rotor. The signal is amplified due to the gear ratio. The signal then travels to the position PID and torque and flux controller (i.e. indirect vector control). The signal generates the required voltage for the 3-phase motor and it is inverted by the PWM. The equations of motion for the motor are then applied. Finally the aerodynamic torque generated from the motor’s velocity is then feedback to the motor. Also, position, velocity, and currents are feedback to have a closed-loop model.

Design of Experiments Method

A typical question is what is design of experiments? To begin, experiments are used to study the performance of processes (shown on Figure 4) or systems. It is typically no
longer cost effective for experiments to be performed in a trial-and-error manner; changing one factor at a time. A far more effective method is to apply a computer-enhanced, systematic statistical-based approach to experimentation, one that considers all factors simultaneously. That approach is called design of experiments (DOE).

Experimental design is a critically important tool in the engineering world for improving the product process. For example, some applications of experimental design in process development and engineering design include:

1. Improved process yields
2. Reduced variability
3. Reduced development time
4. Reduced overall costs
5. Evaluation and comparison of basic design configurations
6. Selection of design parameters so that the product will work well under a wide variety of field conditions; robustness
7. Formulation of new products

The way to perform DOE is problem dependent. However, a common approach is to define an interesting standard reference experiment and then perform new, representative experiments around it (refer to Figure 5). These new experiments are laid out in a symmetrical fashion around the standard reference experiment. Hence, the standard reference experiment is usually called the center-point.

For this study the simulation will have at least eight factors that can be varied in order to characterize and optimize the dynamic test rig. The factors are:

1. Sting inertia
2. Sting spring constant
3. Model inertia
4. Reduced frequency
5. Amplitude of input
6. Noise level added to the position
7. Noise added to the velocity
8. Number of cycles used in calculation

Results and Conclusions

The simulation was run with a 70° delta-wing using the closed-form solution for a slender delta wing. The input signal was a sinusoidal input with amplitude to 1° and with
a pitch oscillation reduced frequency of 0.19 (5.53 rad/s [0.88 Hz]). The testing velocity was 18 m/s. The results presented here are from the simulation. Figure (11) shows the sinusoidal input and the feedback from the motor. It is clear to see that the motor has some phase lag even with feedback controllers. Therefore, the user is not going to obtain the same signal he/she inputted into the system. This is significant because the computational codes that are used to compute the unsteady aerodynamic stability derivatives assume a perfect input.

The electrical torque output is plotted against the aerodynamic torque from the model (i.e. 70° delta wing). The applied aerodynamic torque was time delayed in order to avoid the motor start-up in the beginning of the simulation. The motor electrical torque output will follow a sinusoidal input and adds the aerodynamic torque. This output should be expected as mathematically proven in Equation (5). Also, the electrical torque will have a “noisy” signal due to the pulse-width modulation. This noise can not be removed. Figure (13) illustrates these results.

Since the error between the input and feedback position is small, the velocity plot, Figure (12), will have the input velocity and feedback velocity laying very closely to each other.

Although the unsteady VLM code could not be readily used in the simulation the code was run separately and validated with textbook results from Katz and Plotkin. The pitch oscillation results are shown for various rectangular wing aspect ratios. The moment coefficient is demonstrated on Figure (14) with a wing in 5° pitch oscillation and zero angle of attack. The pitching oscillation can be shown with longer computational time run. The figure does show the initial start-up of the wing and the initial oscillation. The results are varied for different reduced frequency ratios. The lift and drag coefficients have also been calculated, but are not presented here.
After calculation of the total forces and moments the stability derivatives can be calculated and added to the simulation. The best method for calculating stability derivatives is still currently being debated. The study concludes at this last step until a chosen method is implemented.

In conclusion, the paper summarized the importance of this study and noted equations of motion used in the simulation and presented some results. The unsteady vortex lattice code has been completed; results were obtained, and compared with textbook solutions.

**Future Work**

There is quite a lot of future work to be performed. It is listed as follows:

1. Validate the simulation with experimental data.
2. Include the unsteady VLM code.
3. Perform the characterization study using DOE/RSM.
4. Perform the optimization study using DOE/RSM.
5. Apply DOE/RSM to the actual rig itself to improve testing techniques.

**References**


2. Courtesy of Patrick C. Murphy
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Appendix

NOTE: ALL PARAMETERS ARE IN SI-UNIT SYSTEM!!

ALL LOW-PASS FILTERS HAVE BEEN REMOVED
BECAUSE IT ADDS SIGNIFICANT PHASE LAG ..

Author: Brianne Williams