# EARLY WARNING INDICATORS OF ALGAL BLOOMS: EXPECTACTIONS AND OPPORTUNITIES FOR REMOTE SENSING

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### Abstract

Regime shifts, or large changes in state or feedbacks in response to relatively small driver changes, occur in a wide variety of systems from the human body to the global climate. Theory suggests that prior to regime shifts, changes in statistics such as variance and autocorrelation should reflect changes in resilience. In ecology, temporal statistical early warning indicators have been studied in diverse ecosystems and regime shifts, including algal blooms. Prior work suggests there may also be changes in *spatial* statistics before blooms. For this project, I have used a spatial algal bloom model to tested how multiple spatial and temporal statistics change through two algal bloom regime shifts. I found that the best performing statistic and data type (spatial or temporal) was specific to regime shift and depended on driver variable rate of change. These findings will be tested in summer 2019 during a whole-lake fertilization experiment using *in-situ* measurements as well as a novel evaluation of using drone remote sensing to provide the data needed for spatial early warning indicators of algal blooms. With proper validation, emerging technologies like drones and CubeSats offer exciting prospects for applying spatial EWI methods more widely.

## Introduction

Many systems undergo large, seemingly rapid, and unexpected changes, for example the onset of seizers in the brain, collapses of financial institutions, and climate cycles<sup>1</sup>. Collectively, these types of changes are *regime shifts* – large changes in the structure, function, and/or feedbacks of a system in response to relatively small changes in drivers. Regime shifts have been widely studied in diverse ecosystems, including coral reefs, fisheries, drylands, and inland lakes<sup>2</sup>. From a management or societal perspective, the changes associated with regime shifts are often undesirable, making understanding and predicting them a worthwhile goal.

In many ecosystems, regime shifts are also tied to resilience - the ability of a system to recover from perturbation. Dynamical theory suggests that statistical system properties of relevant ecosystem state variables should change in specific ways as resilience decreases and a system approaches a regime shift. These statistics are often referred to as early warning indicators, or EWI. The two most common EWI are changes in variance and autocorrelation as а regime shift is approached<sup>1</sup>. These changes can be used to predict when a system is heading towards a regime shift without exact knowledge of ecosystem driver variables or the thresholds at which regime shifts occur.

Algal blooms can be viewed as a regime shift from clear-water to algae dominated states<sup>3</sup>. Algae are critical components of aquatic ecosystems, and in many systems blooms are a normal part of the seasonal cycle. However, in many lakes, reservoirs, and coastal systems, bloom frequency and severity are increasing<sup>4</sup> and can be indicative of resilience loss. While a number of factors either promote or suppress blooms (e.g. temperature, wind, grazing pressure<sup>5</sup>), the ultimate cause of blooms is most frequently an excess of nutrients.

Aquatic ecosystems provide many important ecosystem services: water for human consumption, agriculture, and industry; harvest of fish and other food products; power; and recreation and other cultural services. The value of these services is large both in terms of directly quantifiable dollar values and indirect benefits<sup>6</sup>. Blooms have large effects on water quality. A study of freshwaters in the US estimated annual costs associated with eutrophication of \$2.2 billion, largely from the impacts of algal blooms<sup>7</sup>. Predicting where and when blooms are likely to occur would be helpful, as it might allow for actions that avoid or minimize negative impacts. For example, advanced warning of blooms in drinking water reservoirs might allow managers to reduce nutrient inputs to prevent a bloom, or at least take a water supply offline while it is treated with algicides. EWI offer a potential way to provide this warning using resilience loss associated with regime shifts.

EWI are expected and have been studied in both temporal and spatial data<sup>8</sup>. Temporal EWI are mostly frequently determined using rolling window statistics, where the EWI statistic (e.g. variance or autocorrelation) are repeatedly calculated on shifting subsets of time series<sup>9</sup>. When a controlling driver is moving towards a regime shift, that change is reflected in the EWI calculated on each sub time series. For spatial EWI, measurements of state variable(s) are taken at several different locations within the ecosystem at approximately the same time. Near regime shift thresholds, low resilience allows shocks to persist and spread spatially, creating patches and increasing spatial variance and autocorrelation.

In aquatic ecosystems there have been fewer studies of spatial EWI relative to temporal EWI. Many early studies on EWI focused on temporal EWI using ecosystem models<sup>10</sup>. These theoretical results were tested using laboratory studies, historical data, and whole-ecosystem experiments<sup>8</sup>. At the ecosystem scale, spatial EWI have been most well studied in terrestrial ecosystems using models of arid vegetation<sup>11</sup>. These models generated hypotheses that have been tested using field experiments and observational remote sensing data<sup>12,13</sup>. In aquatic ecosystems, horizontal spatial heterogeneity is commonly acknowledged but infrequently quantified<sup>14</sup> due to the time and cost of collecting multiple measurements.

The fluid nature of aquatic ecosystem provide some justification mav that measurements at one or a few locations can be representative of an entire ecosystem like a lake, especially if collected over sufficiently long time periods<sup>15</sup>. However, other studies have documented strong "patchiness" in variables ranging from physical to chemical to biological<sup>14</sup>. The success of temporal EWI in aquatic systems noted above suggests either that patchiness is not important in those systems, or that the frequency of data collection or time scales over which data are averaged limit the impact of spatial processes on temporal EWI.

Aquatic spatial EWI studies have mostly been empirical, with initial studies focused on organisms like nekton that largely control their spatial distribution. A few studies have looked at spatial EWI in fish catch data in small lakes<sup>16</sup> and continental shelves<sup>17</sup>, finding that spatial variance increased prior to regime shifts. Recent experimental studies of marine benthic algae communities have found that spatial EWI change predictably near thresholds of algal canopy removal<sup>18</sup>.

Three previous studies have focused on spatial EWI related to algal blooms. The first used a simple eutrophication model, finding that spatial variance increased when nutrient inputs were increased towards a regime shift<sup>19</sup>. While this study provided the first evidence that algal blooms may be preceded by changes in spatial EWI, both the physical and biological components of the model do not include important processes present in real aquatic systems. A second study looked at how spatial EWI changed during an experimentally induced algal bloom, finding large differences in spatial EWI before, during, and after the bloom<sup>20</sup>. This study provided the first empirical evidence that spatial EWI change with algal bloom state. However, it was limited by the relatively coarse temporal resolution of sampling and unknown specifics of the regime shift (e.g. precise bloom timing and regime shift threshold values).

To address some of the limitations of prior studies on spatial EWI in algal blooms, as part of my previous VSGC fellowship I implemented a published spatial algal bloom model<sup>21</sup> with more realistic physical and biological components. I looked at how spatial EWI are expected to change across bloom states and near regime shifts. My work, published last year, found that spatial EWI were distinct for each bloom state and changed near regime shifts using steady-state simulations<sup>22</sup>. It suggested that spatial EWI could be used to classify and compare different aquatic ecosystems for their bloom state and determine relative distance from regimes shifts. However, because simulations were done at steady-state (i.e. nutrient input rate was constant for each simulation), we were not able to conclude that changes in spatial EWI would occur before a bloom when nutrient loading is changing in time. This is an important distinction from a management perspective, where changes in EWI indicating a regime shift could be used to prevent or prepare for an algal bloom.

While there have been experiments looking at changes in both spatial<sup>20</sup> and temporal EWI prior to  $blooms^{23,24}$ , to date there has not been an empirical, direct comparison of these two methods. This summer (2019), our research group will conduct such a comparison in a whole-lake fertilization experiment. To prepare for this ecosystem-scale field experiment, I have built on my previous VSGC fellowship work to generate hypotheses on which EWI indicator(s) will work best. I have done this by adapting the spatial algal bloom model<sup>22</sup> for use with time-varying nutrient loading. Additionally, I have prepared to test the ability of drone-base remote sensing to

collect the spatial data on algal blooms needed for spatial EWI methods.

# Methods

# Spatial algal bloom model

*Model description* - The algal bloom model uses a two dimensional spatial grid (180 x 180 cells) and has two state variables, nutrients (n) and phytoplankton (p), whose dynamics are defined by a pair of differential equations:

$$\partial p = \left[ d\nabla^2 p - (\nabla \cdot vp) + \frac{n}{1+n}p - f_p \frac{p}{1+p} \right] \partial t + p\sigma dW \qquad (1)$$
  
$$\partial n = \left[ d\nabla^2 n - (\nabla \cdot vn) + i_n - a \frac{n}{1+n}p - m_n n \right] \partial t \qquad (2)$$

The first terms in the square brackets of equations 1 and 2 represent diffusion. The second terms represent advection. For nutrients (1), the final three terms represent nutrient inputs, nutrient loss to phytoplankton uptake, and nutrient loss to sinking, respectively. The nutrient input rate  $i_n$  is the main control variable in this study, and its value determines the dynamic state of the model. For phytoplankton (2), the third term in the brackets is growth from nutrient uptake and the last represents losses to zooplankton grazing. The final term in (2) outside the square brackets is random noise, which is added to each grid cell individually to represent random fluctuations and processes not included in the model.

There are three dynamic states occurring at different values of  $i_n$ . At low  $i_n$ the model is in a stable equilibrium state, with temporally constant and spatially uniform (neglecting small noise) concentrations of phytoplankton. As  $i_n$  increases, the stable concentration increases for both nutrients and phytoplankton. Near  $i_n = 0.5$ , the system undergoes a regime shift (Hopf bifurcation) from the low phytoplankton stable state to a stable limit cycle. In this "cycling bloom state", concentrations of nutrients and phytoplankton oscillate in time, and interact with the advective field to create spatial patchiness. The amplitude and baseline of the phytoplankton oscillations grow until in  $\approx 0.9$ , after which the amplitude decreases but the baseline continues to increase. At high *i*<sub>n</sub> the system returns to a stable equilibrium state after going through another regime shift at *i*<sub>n</sub>  $\approx$  1.25, with constant and uniform high values of phytoplankton in this constant bloom stable state.

Simulation framework - To test how EWI changed prior to regime shifts, the model was simulated with time-varying nutrient input rates. Each simulation was started  $0.2 i_n$ units below the regime shift threshold and  $i_n$ increased linearly with each time step to  $0.1 i_n$ units above the threshold. The change in  $i_n$ was set such that the regime shift threshold  $(i_n)$ = 0.5 for the low-input transition and  $i_n = 1.25$ for the high-input transition) was crossed at 2/3 of the total simulation time. In order to assess the impact of rate of change on EWI results, simulations were run for both a slow case (20,000 time units) and fast case (2,000 time units). As each simulation was started and stopped at the same levels of  $i_n$ , this 10x change in total time created 10x faster rate of change in  $i_n$ . Model simulation was carried out using the Euler-Maruyama method with a time step of 0.05. All simulations and calculations were carried out in R.

*EWI calculations* – Temporal and spatial EWI were calculated for each

simulation. Temporal EWI were calculated from 0.1-time unit resolution data from the single grid cell at the center of the grid. This represents data that would be collected from a stationary automated sensor collecting continuous data at the center of a lake (see Discussion). A rolling window size of 100 time units was used for all temporal EWI. The temporal EWI statistics included rollingwindow standard deviation and lag-1 autocorrelation (temporal AC).

Spatial EWI were calculated from 1time unit resolution data on all 180x180 grid phytoplankton values for each spatial snapshot. Analogous statistics where used for spatial EWI for comparison to temporal EWI. Spatial EWI statistics included standard deviation as well as Moran's I (spatial AC). Moran's I is the spatial equivalent of lag-1 temporal autocorrelation; it's the correlation of each grid cell with its 4 neighboring cells for all 180 x 180 cells.

#### Results

Model simulations generated the expected spatial patterns in each system state (Figure 1). For the low-input regime shift that started in the non-bloom stable state, phytoplankton concentrations were relatively uniform, neglecting the added noise, without larger scale patches (Figure 1, left and center



**Figure 1.** Example grids of phytoplankton from the fast simulation case of model going through the low-input regime shift from the low algae stable state to the cycling bloom state. The regime shift occurs at time = 1,333, so time = 1 (left) is in the low algae stable state far from the regime shift, time = 1200 (center) is also in the low algae stable state but close to the regime shift, and time = 1900 is in the cycling state. Color indicates concentration of phytoplankton (red = higher, blue = lower); note independent scaling for each panel.

panels). Closer to the regime shift the range of values observed within a given grid snapshot increased by approximately an order of magnitude. After the regime shift to the cycling state, large-scale patches formed and the range increased even further (Figure 1, right panel). Corresponding changes were seen in the second regime shift, from the patchy, cycling bloom state to the high-algae stable state.

For the slow simulation case (total simulation 20,000 time units), each regime shift was preceded by clear changes in at least one EWI (Figure 2). For the low-input regime shift, changes in spatial EWI were clearest. Spatial SD increased steadily from the start of the simulation (Figure 2E), while spatial AC decreased at first but then increased fairly steadily prior to the regime shift (Figure 2G).

In comparison, temporal SD also increased through the regime shift but with significant fluctuations over shorter time scales (Figure 2A). Temporal AC was highly variable and showed no consistent change prior to the regime shift (Figure 2C).

Temporal SD provided the clearest signal prior to the high-input regime shift for the slow simulation case, with a steady decrease and relatively little variability (Figure 2B). Spatial SD also decreased prior to the regime shift but was highly variable until time 10,000 (Figure 2F). For autocorrelation, spatial AC decreased prior to the regime shift (Figure 2H), while temporal AC showed no change (Figure 2D).



**Figure 2.** EWI results for the slow simulation case. The top row (panels A - D) show rolling-window temporal EWI while the bottom row (panels E - H) show spatial EWI computed from entire grid snapshots. Panels A, C, E, and G are for the low-input regime shift (low-algae stable state to cycling bloom state) and panels B, D, F, and H are for the high-input regime shift (cycling bloom state to high-algae stable state). Red vertical lines indicate time of the regime shift.



**Figure 3.** EWI results for the fast simulation case. Red vertical lines indicate time of the regime shift.

For the fast simulation case (total simulation time of 2,000 time units), EWI results were largely the same for the low-input transition as for the fast simulation. Spatial SD (Figure 3E) and spatial AC (Figure 3G) had the clearest and most consistent trends prior to the regime shift, while temporal SD generally increased but with lots of variability (Figure 3A) and there was no discernable change in temporal AC (Figure 3C).

The temporal EWI were also similar for the high-input regime shift of the fast simulation case. Temporal SD declined steadily through the transition (Figure 3B) and there was no clear change in temporal AC (Figure 3D). Spatial EWI were not the same for the fast simulation compared to the slow simulation case. Spatial SD declined and was highly variable through the transition (Figure 3F), but did not become constrained at low values prior to the regime shift as in the slow simulation case (Figure 2F). Spatial AC did not fall appreciably until after the regime shift (Figure 3H), in contrast with the slow simulation case (Figure 2H).

#### **Discussion**

Results from this study show that changes in resilience in aquatic ecosystems prior to algal blooms may be preceded by changes in EWI statistics. This study provided the first direct comparison of multiple temporal and spatial EWI using a spatial algal bloom model that incorporates realistic physical and biological processes. Our findings suggest that the best performing EWI is likely to be regime shift specific and dependent on the rate at which the regime shift is approached.

The better performance of the spatial EWI relative to temporal EWI at the low-input regime shift is likely due to interactions between small-scale variability and advection. Random variations from stochasticity create small patches of higher or lower phytoplankton concentrations in the low-algae stable state, which get moved around the grid by advection. These patches moving through the single location in the center of the grid create temporal variability. Spatial EWI, in contrast, use all the grid phytoplankton concentrations in each calculation, so this small-scale variability is included and does not change. For the highinput regime shift, spatial SD and spatial AC did not perform as well as temporal SD. The mechanism behind this difference is not as clear, but is likely related to increased patch size and cycle amplitude in this state.

The model used in this study (like all models) is a simplification of reality and cannot capture all important processes. For example, in real aquatic ecosystems, phytoplankton will be made up of several or many species that interact with each other and other ecosystem components, vs. a single phytoplankton state variable. Additionally, advection is likely to be time-varying and related to e.g. wind speed and direction, vs. temporally constant. While these processes could be incorporated in a more complex model, simple models that include relevant processes and recreate observed dynamics are useful to establish expectations for later empirical studies. This approach, modeling studies followed by empirical tests, has proven fruitful for studies of temporal EWI in aquatic ecosystems (e.g. $^{10,25}$ ).

Our research group will test the findings of this work this summer, directly comparing spatial EWI and temporal EWI in an experimentally induced regime shift for the first time. The experiment is occurring at the University of Notre Dame Environmental Research Center (UNDERC) in the Upper Peninsula of Michigan. We will collect intensive time series and spatial data while fertilizing Peter Lake daily with inorganic nitrogen and phosphorus to cause an algal bloom. Continuous time series of temperature, conductivity, pH, dissolved oxygen (DO), chlorophyll, and blue-green algae (BGA) will be recorded every 5 minutes using automated sensors attached to a buoy at the center of the lake. These data will be used for rollingwindow temporal EWI analyses. The first source of spatial data will be the FLAMe (Fast Limnology system Automated Measurements; flame.wisc.edu), which was developed by our collaborators at the University of Wisconsin-Madison<sup>14</sup>. This system is made up of several sensors contained within a portable box along with a GPS, a water pump, and water intake and exhaust apparatus mounted to a boat. When underway, water is pumped through the sensor box, and high frequency (1 Hz) measurements on water parameters and GPS location are taken. These in-situ water quality and GPS measurements are converted to maps for visualization and used for calculation of spatial EWI.

This study will allow, for the first time, direct comparison at relatively high frequency of whether spatial or temporal EWI provide a more reliable warning prior to a real-world bloom. This analysis will be done qualitatively by plotting temporal and spatial EWI to compare how early each indicator changes prior to the bloom, the magnitude of that change, and consistency of the change (is there a clear signal of resilience change relative to background variability?). The indicators will also be compared quantitatively using rolling window Spearman rank correlation to test the strength of trends in EWI.

Finally, the detailed spatial data collected by the FLAMe system will also enable a novel test of remote sensing (RS) to measure algal blooms in both time and space. Remote sensing, most commonly satellite RS, is an alternative to wet chemistry and in-situ mapping that can collect large amounts of bloom data. Satellite RS, though, has tradeoffs between revisit frequency and spatial resolution. Drone-based RS is a relatively new research technique that has the potential to provide both high resolution (cm) and frequency (up to minutes or hours). However, most drone RS studies of algal blooms have

been limited to one or a few ecosystems and sampling events, leaving questions about the method's broad applicability<sup>26</sup>.

We will conduct experiments and collect data to address three novel objectives: can drone RS capture changes in bloom conditions 1) in individual lakes through time, 2) spatially within a lake, and 3) across different lakes? I will compare drone-derived chlorophyll concentrations to wet chemistry and in-situ fluorometric measurements. Data will be collected several times per week in Peter Lake concurrent with FLAMe mapping and in a survey of 30 lakes spanning ranges of dissolved eutrophication and carbon concentrations. Success would provide one of the most comprehensive demonstrations to date of the utility of drone RS for measuring algal blooms.

Looking ahead, the findings of this work are promising in light of emerging technologies. Inexpensive and easy-to-use of drones and constellations of small satellites (CubeSats) have the potential to provide both high frequency and resolution data with wide coverage. Such data would be useful in both applied contexts (e.g. detecting and treating blooms before they grow and spread) as well as for basic research (e.g. enabling studies on spatial resilience indicators).

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